

Íslenska stærðfræðafélagið
Félag raungreinakennara í framhaldsskólum

30th of September 2025
Time limit: 2 ½ hours.

Stærðfræðikeppni framhaldsskólanema 2025–2026

Full Name: _____

Kennitala: _____ Phone number: _____

Email address: _____

School: _____ Class: _____

Did you start your high school studies 2024 or earlier?

☐ Yes (*Upper division*) ☐ No (*Lower division*)

I	
II	
16	
17	
18	
19	
Σ	

Those who started high school 2024 or earlier compete in the *upper division*. Others compete in the *lower division*. The top scoring students in each division are invited to compete in the finals.

Instructions:

1. Do not open this booklet before you are told to do so.
2. This is **not a test**. Answer the questions you can and do not worry about skipping some questions. Some of the questions are very hard.
3. In solutions to the last four problems, those in section 3, you are expected to give a complete explanation for how the answer was found. Write down a finalised solution, drafts belong on scratch paper. When awarding points consideration is given to the accuracy of the reasoning and the clarity of the presentation.
4. Pictures that accompany certain problems are only meant for clarification. It is not guaranteed that they are drawn in the correct proportions.
5. Permitted items are writing utensils, a compass and a ruler. **The use of calculators and phones is prohibited.**

6. You are given that $x + \frac{1}{x} = 45$, what is the value of $x^2 + \frac{1}{x^2}$?

☐ 2023

☐ 2024

☐ 2025

☐ 2026

7. You draw two cards from a normal deck of cards with 52 cards. What is the probability that you draw the cards in strictly increasing order? The order goes ace, two, three, ..., ten, jack, queen, king. Strictly increasing order means the second card is strictly greater than the first. The first card is not returned to the deck before the second is drawn.

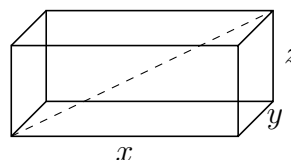
☐ 6/13

☐ 24/51

☐ 25/52

☐ 1/2

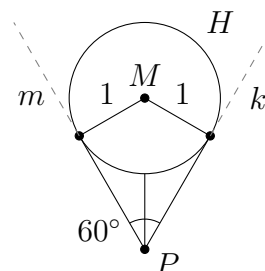
8. A box with all right angles has length x , width y and height z . What is the length of the line segment going from one corner to the opposite one?


☐ $\sqrt{x^2 + y^2 + z^2}$
☐ $\sqrt{\sqrt{x^2 + y^2} + z^2}$
☐ $\sqrt{x^2 + y^2} + z^2$
☐ $\sqrt{x^2 + y^2} + z$

9. Which of the following values do not end with the digit 0?

☐ $7!$
☐ $17^7 - 7^7$
☐ $17^7 + 7^7$
☐ $11^7 - 1$

10. A circle H has radius 1 and centre M . The tangents to the circle, m and k , intersect at a point P outside the circle. The angle between the tangents is 60° . What is the shortest distance from P to the circle?


☐ $2\sqrt{2} - 2$
☐ $2/\sqrt{3}$
☐ $\pi/3$
☐ 1

Second part (30 points)

In this section there are 5 problems, each worth 6 points. Write your answer on the line. Do not give reasoning for your answer. For an incorrect, incomplete or ambiguous answer no points are awarded.

Answers in this section are positive integers smaller than 100.000.

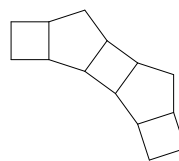
- 11.** There exists exactly one three digit number which becomes 4.5 times bigger when the first and third digit are swapped. What number is that?

Answer: _____

- 12.** What is the value of $\sqrt{42 + \sqrt{42 + \sqrt{42 + \sqrt{42 + \dots}}}}$?

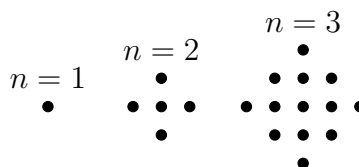
Answer: _____

- 13.** Squares and regular pentagons are arranged into a circle like the section to the right shows. How many squares and pentagons do you need in total to make a full circle?



Answer: _____

- 14.** On the image to the right we see a pattern of points for $n = 1, 2, 3$. How many points will there be in the pattern for $n = 50$?



Answer: _____

- 15.** Find the largest number x such that 2^x divides $(2^{2^{2^2}} + 2)!$.

Answer: _____

Third part (40 points)

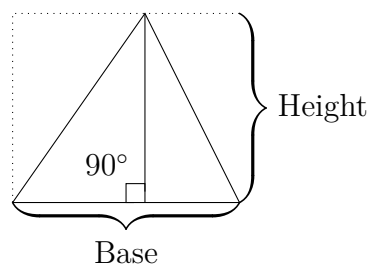
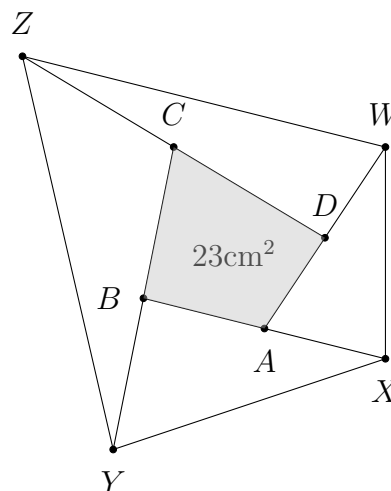
In this section there are four problems, each worth 10 point. **You must give reasoning for your answers in this section.** When awarding points considerations are given to structure, precision and clarity in the presentation of the solution. Note that points are awarded for a partial solution to the problems or for coming up with an idea which is an important part of a solution.

Problem 16

If the number 2025 is split into two two-digit numbers we get 20 and 25. They sum to 45 and $45^2 = 2025$. The next number with this property is 3025, what four digit number with this property is next?

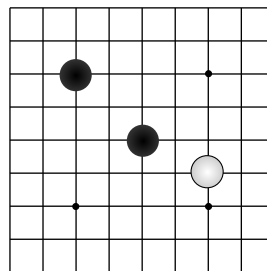
Problem 17

You are given a quadrilateral $ABCD$ with area 23cm^2 . Its sides are extended to be twice as long, as shown on the picture. This gives us four new points X, Y, Z, W . What is the area of the quadrilateral $XYZW$? Note that the area of a triangle can be calculated as its base times its height divided by two, see the image below.



Problem 18

Jörmunrekur has a large $n \times n$ Go board and enough black and white stones to place on it. He wants to place $n \times n$ stones on the board such that each stone has exactly two neighbours in its own colour. Two stones are considered neighbours if they are adjacent (horizontally or vertically) with no gap in between. For which n is this possible? (The image is of a 9×9 board).



Problem 19

We have a sequence of numbers $a_1, a_2, a_3, \dots$

You are given that

$$a_{2n} = a_n, \quad a_{4n-3} = 2a_{2n-1} - 1, \quad \text{and} \quad a_{4n-1} = 2a_{2n-1} + 1$$

for all $n = 1, 2, \dots$. Calculate the value of $a_1 + a_2 + \dots + a_{2025}$.

Partial points are awarded for determining the sequence $a_1, a_2, a_3, \dots$